COMPLEX HEAT EXCHANGE IN THE CASE OF THE LAMINAR FLOW OF A SCATTERING MEDIUM IN A CYLINDRICAL CHANNEL

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Complex heat exchange in the case of the laminar motion of a nonscattering medium in a cylindrical channel has been discussed in [1, 2]. In [2] the channel is taken to be infinite, and the temperature of the lateral surface at x = 0 varies discontinuously. In [1] heat exchange is investigated in a channel of finite length. Let us adopt the assumptions of [1], but we will assume the medium to be absorbing and scattering.

Let us assume that at x = 0 (Fig. 1) a medium with temperature T_0 and a parabolic velocity distribution flows into a cylindrical channel whose walls are at the constant temperature T_W. At x = L the channel is closed by a black permeable membrane at temperature T_W. The energy equation is of the form

$$\mathrm{Pe}\, V(\eta)\frac{\partial\Theta}{\partial\xi} = 4\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\Theta}{\partial\eta}\right) - \frac{D^2}{\lambda T_0}\operatorname{div}\mathbf{q}_r, \quad (1)$$

where $\mathrm{Pe} = \rho c_p \langle v\rangle D/\lambda$ is the Péclet number; $\langle v\rangle$ is the average velocity; $V = v/\langle v\rangle$; $\xi = x/D$; $\eta = 2r/D$; $\Theta = T/T_0$; D is the channel diameter; ρ is the density of the medium; c_p is its specific heat; v is the local velocity; T is the temperature; r and x are the radial and longitudinal coordinates; λ is the thermal conductivity coefficient; and $\mathbf{q}_r$ is the radiation flux density vector. The boundary conditions for Eq. (1) have the form

$$\Theta(\xi = 0) = 1, \ \Theta(\eta = 1, \ \xi > 0) = \Theta_w, \ \partial\Theta/\partial\eta|_{\eta=0} = 0.$$

The divergence of the radiative flux was calculated with the radial and longitudinal temperature distribution in the channel taken into account. Scattering is taken into account in the quasi-one-dimensional approximation suggested in [3]. It is assumed in this approximation that scattering occurs only forward and backward along the ray path, but the geometry of the medium is taken completely into account. The solution for the Green's function of the one-dimensional radiative transfer problem was obtained in advance and has the form

$$I^+ = I_0^+ e^{-\varkappa(\tau-\tau_1)}\frac{1 - R_\infty^2 e^{-2\varkappa(\tau_0-\tau)}}{1 - R_\infty^2 e^{-2\varkappa(\tau_0-\tau_1)}}, \quad (2)$$

$$I^- = R_\infty I_0^+ e^{-\varkappa(\tau-\tau_1)}\frac{1 - e^{-2\varkappa(\tau_0-\tau_1)}}{1 - R_\infty^2 e^{-2\varkappa(\tau_0-\tau_1)}}$$

for the point $\tau > \tau_1$ (Fig. 2).

The self-radiation of the point τ_1 is unitary and isotropic. If τ lies in $(0, \tau_1)$, then the solution for the Greens function is obtained from (2) by a 180° rotation of the τ axis

$$I^- = I_0^- e^{-\varkappa(\tau_1-\tau)}\frac{1 - R_\infty^2 e^{-2\varkappa\tau}}{1 - R_\infty^2 e^{-2\varkappa\tau_1}}, \quad (3)$$

$$I^+ = R_\infty I_0^- e^{-\varkappa(\tau_1-\tau)}\frac{1 - e^{-2\varkappa\tau}}{1 - R_\infty^2 e^{-2\varkappa\tau_1}},$$

where τ_0, τ, and τ_1 are the products of the absorption coefficient by the appropriate length, $\varkappa = \sqrt{(1-\gamma)(1-\mu\gamma)}$; $R_\infty = (\sqrt{1-\mu\gamma} - \sqrt{1-\gamma})/(\sqrt{1-\mu\gamma} + \sqrt{1-\gamma})$; γ is the ratio of the scattering coefficient to the absorption coefficient k, and μ is the average cosine of the scattering angle affiliated with an elementary scattering event. The co-

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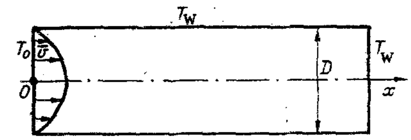

Fig. 1

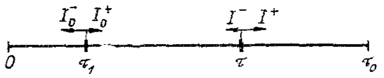

Fig. 2

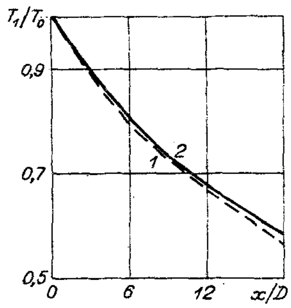

Fig. 3

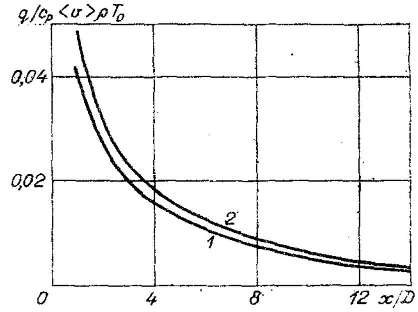

Fig. 4

efficients k, γ, μ can be calculated from the Mie theory [4]. The quantities I_0^+ and I_0^- are the intensities of the resulting radiation at the point τ_1. The expressions

$$I_0^+ = \frac{1+R_1}{1-R_1R_2}, \qquad I_0^- = \frac{1+R_2}{1-R_1R_2}$$

are obtained for them, where

$$R_1 = R_\infty \frac{1-e^{-2\varkappa\tau_1}}{1-R_\infty^2 e^{-2\varkappa\tau_1}}; \qquad R_2 = R_\infty \frac{1-e^{-2\varkappa(\tau_0-\tau_1)}}{1-R_\infty^2 e^{-2\varkappa(\tau_0-\tau_1)}}.$$

The quasi-one-dimensional approximation was used in [5, 6]. The problem of radiative–conductive heat exchange in a plane layer is solved in [5], and good agreement of the results with the exact numerical solution is shown. This approximation is more accurate the more the scattering indicatrix is elongated along the ray. When scattering is not taken into account it leads to the exact solution, and the results in the case of a purely scattering medium [5, 6] are in good agreement with the results of the exact calculations of the spherical albedo of a layer in the case of a spherical scattering indicatrix [7].

With the use of the Green function (2) and (3) the divergence of the radiative flux vector is determined in dimensionless form by the expression

$$\frac{D^2}{\lambda T_0}\operatorname{div}\mathbf{q}_r = \frac{\mathrm{Bu}(1-\gamma)}{\mathrm{Bo}\,\mathrm{Pe}^{-1}}\left\{4\Theta^4 - 2\int_{-\pi/2}^{\pi/2}\int_0^{\pi}\left[B(I^+ + I^-) + \right.\right.$$
$$\left.\left. + \frac{\mathrm{Bu}(1-\gamma)}{\pi}\int_0^{S_0}\Theta^4(S)(I^+ + I^-)\,dS\right]\sin\varphi\,d\varphi\,d\psi\right\},$$

where $\mathrm{Bu}=kD$; $\mathrm{Bo}^{-1}=\sigma T_0^3/c_p\rho\langle v\rangle$; σ is the Stefan–Boltzmann constant; B is the effective radiation density of the wall relative to σT_0^4; S is the ratio of the distance between the emitting point and the point at which div $\mathbf{q}_r$ is determined to the diameter of the cylinder; and S_0 is the analogous ratio for the case of the bounding emittin points.

The energy Eq. (1) is solved numerically by the finite-difference method with the use of the sweep metho in combination with an iterative process. The quantity div $\mathbf{q}_r$ was determined from the derived field H, and then Eq. (1) was solved and a new temperature field was found. The process converged after approximately six iterations. The case of the cooling of a medium upon its flow along a channel is considered ($T_w < T_0$). The calculation was performed on a BÉSM-6 computer with the following parameters: $\Theta_w = T_w/T_0 = 0.18$, Pe = 1000, $\gamma = 0.5$, $\mu = 0.7$, Bo = 29.4, L/D = 18, and for different values of the optical thickness of the medium Bu. The walls were assumed to be black. Comparison of the results of the calculation in scattering and nonscattering media was carried out for identical values of the absorption coefficient of the medium. When scattering is taken into account, the temperature of the medium decreases more slowly along the channel than when scattering is not taken into account. This result pertains to both the axial local temperature and the average temperature over the cross section.

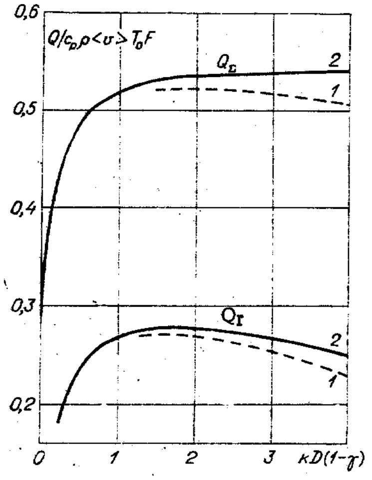

Fig. 5

The temperature distribution on the cylinder axis without and with scattering taken into account is given in Fig. 3 (curves 1 and 2, respectively) for $kD(1-\gamma)=4$. In the initial section of the channel the temperature gradient at the wall is lower in a scattering medium than for a nonscattering one, and then the temperature gradient becomes larger in the scattering medium than in the nonscattering one as the medium moves along the channel. The conductive component of heat exchange varies in a corresponding manner. The local resultant flux of radiation onto the wall is also less in the scattering medium in the initial section, and it can be larger in the final section than for a medium without scattering.

The variation along the channel length of the density of the resultant radiation onto the wall is shown in Fig. 4 both for a scattering and a nonscattering medium (curves 1 and 2, respectively). The comparison is made with a value for the product of the absorption coefficient by the diameter of $(1-\gamma)\mathrm{Bu}=4$. A smaller amount of heat is transmitted to the wall of the channel in a scattering medium, and the temperature of the medium at the exit has a higher value than in the analogous case without scattering taken into account.

It is interesting to trace the effect of scattering on the total heat exchange in the channel. The dependence of the total heat flux Q_Σ and the radiative flux Q_r transmitted to the side wall of the entire channel on the product $(1-\gamma)\mathrm{Bu}$ is shown in Fig. 5 (curves 1 are with scattering taken into account, and curves 2 are without scattering taken into account; $F=\pi D^2/4$). The results show that the effect of scattering is insignificant at small and moderate optical thicknesses of the medium, but it increases as the optical thickness increases. It follows from Fig. 5 that there exists in a scattering medium a maximum heat transmission at some optical thickness of the medium, and for the radiative component the position of the maximum is shifted into the region of lower values of the optical thickness. It is shown in [1] that the quantity Q_Σ can have a maximum without scattering taken into consideration. The presence of the maximum is explained by the fact that as the absorption coefficient increases the optical thickness of the relatively cold boundary layer, which screens the radiation arriving at the wall from the hot inner layers, increases. The screening role of the boundary layer increases in a scattering medium.

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